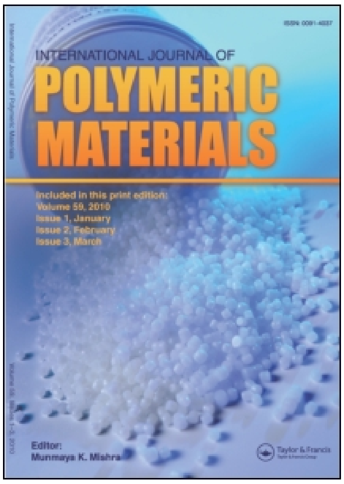

International Journal of Polymeric Materials
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713647664

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To cite this Article Nakamura, K. , Imada, K. and Takayanagi, M.(1974) 'Analyses of Solid State Extrusion Process of Polymeric Materials by Pressure Dependent Yield Criteria', International Journal of Polymeric Materials, 3: 1, 23 — 28
To link to this Article: DOI: 10.1080/00914037408072372
URL: http://dx.doi.org/10.1080/00914037408072372

Intern. J. Polymeric Mater., 1974, Vol. 3, pp. 23–28

Printed in Reading, England

Analyses of Solid State Extrusion Process of Polymeric Materials by Pressure Dependent Yield Criteria

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(*Received April 3, 1973*)

Among various methods to estimate extrusion pressure in solid state extrusion, Sachs's method[1] is one of the most convenient and widely used ones. According to this method the equation for extrusion pressure is derived by taking accounts of the equilibrium of the forces in the tapered die part and of the yield criterion of the material being extruded. Analytical solutions have easily been obtained in the cases that the yielding behavior of the material obeys Tresca or von Mises yield criterion and where the yield stress does not depend on strain or the strain dependence of yield stress is represented in a simple analytical form. The yielding behavior of metallic materials satisfies these conditions.

On the other hand the yield stress of polymeric materials increases rapidly with increasing strain owing to the orientation of molecular chains in the deformation process. For example polyethylene and nylon 6 have been reported to show such a strain-hardening phenomena in uniaxial drawing that the relation between true stress and true strain is hyperbolic in a log-log plot.[2] Imada *et al.*[3] have extended Sachs's method to materials which show strain hardening behavior, and they derived Eq. (1) to estimate the extrusion pressure.

$$P = -\sigma_x = (1 + B)\int_0^{2ln(R/r)} Y(\epsilon) \cdot \exp(B\epsilon)\sigma \cdot d\epsilon - \sigma_{xo} \cdot \left(\frac{R}{r}\right)^{2B} \quad (1)$$

where $Y(\epsilon)$ is a function representing the strain-hardening behavior of polymeric materials, R and r are inlet and outlet radii of the die, respectively,

σ_{xo} is the stress at the die exit and B is a parameter given by Eq. (2).

$$B = \mu \cdot cot\alpha \tag{2}$$

where μ is a frictional coefficient between the material and the die and α is a half-die angle. Eq. (1) has been found to agree with the experimental results relatively well for polyethylene,[3,4] nylon 6[3] and polypropylene.[5] When deriving Eq. (1) it was assumed that the material obeys the Tresca or the von Mises yield criterion. In recent years, however, the yielding behavior of polymeric materials has been confirmed to be affected by hydrostatic stress from testing under various combined stress conditions and under hydrostatic pressure.[6] Concomitantly Hu and Pae[7] and Sternstein[8] have proposed new yield criteria which to the classical Coulomb yield criterion[9] take account of the effect of hydrostatic pressure in addition. Application of these criteria to the yielding behavior of polymeric materials has been discussed experimentally by Mears *et al.*[10,11] In this report we propose new equations for the extrusion pressure in solid state extrusion, which can be derived from the Coulomb and the Sternstein yield criteria.

Coulomb yield criterion is expressed as follows.

$$(\tau + \mu_i\sigma)_{\max} = \tau_c \tag{3}$$

where τ and σ are shear and normal stresses acting on any plane in the material, respectively, τ_c is a critical shear stress called the "cohesion" of the material and μ_i is a "coefficient of internal friction". Equation (3) is rewritten in terms of principal stresses σ_1, σ_2 and σ_3 ($\sigma_1 \geq \sigma_2 \geq \sigma_3$).[12]

$$\frac{1}{2\cos\theta}\left\{(\sigma_1 - \sigma_3) + \sin\theta \cdot (\sigma_1 + \sigma_3)\right\} = \tau_c \tag{4}$$

where

$$\tan\theta = \mu_i \tag{5}$$

Now if we assume that Eq. (4) is applicable not only to initial yielding but also to the yielding behavior of the material strained to ϵ and that τ_c is a function of strain ϵ while μ_i is constant throughout deformation, we can estimate τ_c from uniaxial extension data. In uniaxial tensile deformation $\sigma_1 = Y(\epsilon)$ and $\sigma_2 = \sigma_3 = 0$. Thus,

$$\tau_c = \frac{1 + \sin\theta}{2\cos\theta} Y(\epsilon) \tag{6}$$

From Eqs. (4) and (6) the following expression for yield criterion becomes possible.

$$(\sigma_1 - \sigma_3) + \sin\theta \cdot (\sigma_1 + \sigma_3) = (1 + \sin\theta) \cdot Y(\epsilon) \tag{7}$$

Next we apply this yield criterion to solid state extrusion process, in which the state of stress can be described as follows.

$$\left.\begin{aligned}\sigma_1 &= \sigma_x \\ \sigma_2 = \sigma_3 &= \sigma_r\end{aligned}\right\} \quad (\sigma_r < \sigma_x < 0) \tag{8}$$

where σ_x and σ_r are the stresses along axial and radial directions, respectively. Substitution of Eq. (8) into Eq. (7) results in

$$\sigma_r = A\{\sigma_x - Y(\epsilon)\} \tag{9}$$

where

$$A = (1 + \sin\theta)/(1 - \sin\theta)$$

From the equilibrium of the forces acting on the infinitesimal element in the material being extruded,[1] we obtain

$$\frac{d\sigma_x}{d\epsilon} = \sigma_x - \sigma_r \cdot (1 + B) \tag{10}$$

where B is a constant defined with Eq. (2). From Eqs. (9) and (10) we obtain

$$\frac{d\sigma_x}{d\epsilon} = \{1 - A \cdot (1 + B)\} \cdot \sigma_x + A \cdot (1 + B) \cdot Y(\epsilon) \tag{11}$$

$$= (1 + \beta) \cdot Y(\epsilon) - \beta \cdot \sigma_x \tag{12}$$

where

$$\beta = A \cdot (1 + B) - 1 = \frac{1 + \sin\theta}{1 - \sin\theta}(1 + B) - 1 \tag{13}$$

Equation (12) can easily be solved to result in Eq. (14).

$$\sigma_x = -(1 + \beta)\int_0^{\epsilon_o} Y(\epsilon) \cdot \exp(\beta\epsilon) \cdot d\epsilon + \sigma_{xo} \cdot \exp(\beta\epsilon_o) \tag{14}$$

By considering that $\epsilon_o = 2ln(R/r)$, which is a strain at die exit, the equation for extrusion pressure can be expressed as follows.

$$P = -\sigma_x = (1 + \beta)\int_0^{2ln(R/r)} Y(\epsilon) \cdot \exp(\beta\epsilon) \cdot d\epsilon - \sigma_{xo} \cdot \left(\frac{R}{r}\right)^{2\beta} \tag{15}$$

Now we can recognize that Eq. (15) has the same form as Eq. (1). Although B in Eq. (1) and β in Eq. (15) must be determined from parameters μ, α and θ, determination of μ is very difficult. In our previous papers[3–5] we regarded μ as an adjustable parameter and the value of B or μ was determined so that the calculated values of extrusion pressure were in best accord with the observed values. Since Eq. (15) is formally the same as Eq. (1), the calculated values of extrusion pressure obtained by setting adjustable parameter β to be equal to B value reported previously[3–5] can also satisfy the observed values.

By using the value of β thus obtained, a new value of B can be determined by Eq. (13), and further a new value of μ by Eq. (2). In order to determine B by Eq. (13) the value of $\sin\theta$, which is associated with the coefficient of internal friction μ_1, is necessary. The approximate evaluation of $\sin\theta$ is possible from the data of initial tensile yield stress σ_{to} and initial compressive yield stress σ_{co} with Eq. (16).

$$\sin\theta = -\frac{\sigma_{to} + \sigma_{co}}{\sigma_{to} - \sigma_{co}} \tag{16}$$

For one example we have measured σ_{to} and σ_{co} of polypropylene at 70, 100 and 130°C and estimated the μ value. Table I lists μ values evaluated by Eq. (1),[5] the values of σ_{to}, σ_{co}, $\sin\theta$ and μ evaluated by Eq. (15) for polypropylene.

Sternstein yield criterion as represented by Eq. (17) is a modified form of the von Mises criterion.

$$\tau_{oct} + \mu' \cdot \sigma_m = \tau_o \tag{17}$$

where τ_{oct} is octahedral shear stress, σ_m is mean normal stress, τ_o is octahedral shear stress at zero pressure and μ' is a constant of material. τ_{oct} and σ_m are expressed in terms of three principal stresses by Eqs. (18) and (19), respectively,

$$\tau_{oct} = \tfrac{1}{3}\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\}^{1/2} \tag{18}$$

$$\sigma_m = \tfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \tag{19}$$

Now by assuming that μ' is a constant and τ_o is a function of strain ϵ and by representing τ_o in terms of $Y(\epsilon)$ by the same method as described above, Eq. (20) can be obtained as an equation corresponding to Eq. (7).

$$\begin{aligned}\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\}^{1/2} + \mu' \cdot (\sigma_1 + \sigma_2 + \sigma_3)\\ = (\sqrt{2} + \mu') \cdot Y(\epsilon)\end{aligned} \tag{20}$$

Substitution of Eq. (8), which describes the state of stress in solid state extrusion, into Eq. (20) gives

$$\sigma_r = C \cdot \{\sigma_x - Y(\epsilon)\} \tag{21}$$

where

$$C = (\sqrt{2} + \mu')/(\sqrt{2} - 2\mu')$$

Eq. (21) has the same form as Eq. (9) except that A of Eq. (9) is replaced by C. Thus the subsequent analytical method becomes the same as in the case of Coulomb criterion, and finally Eq. (22) is obtained as an equation for estimating extrusion pressure.

$$P = -\sigma_x = (1 + \beta')\int_0^{2ln(R/r)} Y(\epsilon) \cdot \exp(\beta'\epsilon) \cdot d\epsilon - \sigma_{xo} \cdot \left(\frac{R}{r}\right)^{2\beta'} \tag{22}$$

where

$$\beta' = C \cdot (1 + B) - 1 = \frac{\sqrt{2} + \mu'}{\sqrt{2} - 2\mu'}(1 + B) - 1 \tag{23}$$

Eq. (22) shows that the analytical method by using Sternstein yield criterion leads to the same result as Eq. (1) or Eq. (15) as far as the form of equation is concerned. The difference among Eqs. (1), (15) and (22) results in the difference of the value of adjustable parameter μ. The μ values thus obtained are necessary to be verified for their adequateness. The constant μ' in the Sternstein yield criterion can also be estimated approximately by using Eq. (24).

$$\mu' = -\sqrt{2}\frac{\sigma_{to} + \sigma_{co}}{\sigma_{to} - \sigma_{co}} \tag{24}$$

The values of μ' and μ evaluated by Eqs. (24) and (22), respectively, are listed in Table I for polypropylene.

TABLE I

The values of μ from Eqs. (1), (15) and (22), $\sin\theta$ and μ' evaluated by using tensile and compressive yield stress for polypropylene at 70, 100 and 130°C

Temperature °C	μ from Eq. (1)	σ_{to} Kg/mm^2	σ_{co} Kg/mm^2	$\sin\theta^a$ Eq. (16)	μ from Eq. (15)	μ'^b Eq. (24)	μ from Eq. (22)
70	0.30	1.94	2.12	0.044	0.24	0.063	0.22
100	0.30	1.13	1.39	0.097	0.18	0.137	0.12
130	0.30	0.48	0.64	0.143	0.13	0.202	0.05

[a] $\sin\theta$ was estimated by using Eq. (16).
[b] μ' was estimated by using Eq. (24).

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